Generalized Skew Reverse Derivations in Semi Prime Rings

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***Abstract* :** The study of generalized skew reverse derivations in semiprime and prime rings. In section 1 we study different situations for a semi prime ring to be commutative by admitting generalized skew reverse derivations, several authors proved commutative property by admitting reverse derivations, generalized reverse derivations in semi prime rings and prime rings. In this line of investigation we extend the results of Ali et al [2] where they studied the results on lie ideals with multiplicative derivations in prime and semiprime rings to semi prime rings admitting generalized skew reverse derivations. In this paper, we extend the results of Chirag Garg and Sharma [9], where they studied on some identities by admitting generalized (α,β) derivations on a non zero square closed Lie Ideals of a prime ring to generalized skew reverse derivations on Ideals in prime rings.

Keywords : Prime rings, Generalized skew reverse derivations, semiprime rings, skew reverse derivations.

1. INTRODUCTION

Throughout, R denotes a ring with center $Z(R)$. We write $[x,y]$ for $xy - yx$. Recall that a ring R is called *prime* if $aRb = 0$ implies $a = 0$ or $b = 0$; and it is called *semiprime* if $aRa = 0$ implies $a = 0$. A prime ring is obviously semiprime. An additive mapping d from R into itself is called a *derivation* if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. a mapping f from R into itself is *commuting* if $[f(x),x] = 0$, and *skew commuting* if $f(x)x + xf(x) = 0$, for all $x \in R$. A considerable amount of work has been done on derivations and related maps during the last decades (sec. eg., (Beidar et al[4], Bell and Martindale[5], Bresar[6]) and references therein). Bresar and Vukman [7]) have introduced the notion of a *reverse derivation* as an additive mapping d from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$. for all $x, y \in R$. Obviously, if R is commutative, then both derivation and reverse derivation are the same. In the present note, we explore more about reverse derivations. We will show that while the notions of derivation and reverse derivation do not coincide, the set of all derivations and the set of all reverse derivations on a ring R are not disjoint. Recall that a ring R is called *anticommutative* if $ab + ba = 0$ for all $a, b \in R$. We will provide some properties for reverse derivations on anticommutative rings. On the way of studying derivations and reverse derivations, we will show that if d is a skew commuting derivation on a non-commutative prime ring, then d must be trivial. One of our main aims is to show that for a semiprime ring R, any reverse derivation is in fact a derivation mapping R into its center. This, in turn, will force a prime ring with a non-trivial reverse derivation to be commutative. For example, we refer the readers (Ali and Dar[1], Ali [3], Chaung [8], Lee[10], Oukhtite and Mamouni[11], Rehman et al[12], Samman[13] and Tiwari et al[14]) have been analysed the generalized reverse derivation in semiprime rings and where further references can be found). For motivation and a close view on reverse derivations, we provide the following examples.

2. GENERALIZED SKEW REVERSE DERIVATIONS IN SEMIPRIME RINGS

All through the section R is a semiprime ring. A ring R is said to be semiprime if for any x in R, $xRx = 0$ implies $x = 0$. An additive map g: R R is said to be skew reverse derivation if $g(xy) = g(y)x + \alpha(y)g(x)$, for all $x, y \in R$ associated with an automorphism of R and an additive map $F : R \to R$ is said to be generalized skew reverse derivation if $G(xy) = G(y)x + \alpha(y)g(x)$, for all $x, y \in$ R associated with skew reverse derivation d and an automorphism α of R.

Lemma 1.1 : (Ali et al.[2], Lemma 2.1) : If R is a semiprime ring and I is an ideal of R, then I is a semiprime ring.

Theorem 1.1. : R is a semiprime ring, I is a nonzero ideal of R and (G, g) is a generalized skew reverse derivation associated with and automorphism α such that $G[u,v] = \pm(uv + vu)$, for all $u, v \in I$, then $[g(u), \alpha(u)] = 0$.

Proof: Firstly, we consider

$$G[u, v] = uv + vu, \text{ for all } u, v \in I. \qquad 2.1$$

Replacing v by uv in equation 2.1, then $g[u, uv] = uuv + uvu,$ for all $u, v \in I.$

$$u^2 v + uvu = g\{u[umv]\}.$$

$$= G[u, v]u + \alpha[u, v]g(u).$$

$$= (uv + vu)u + \alpha[u, v]g(u).$$

$$= uvu + vu^2 + \alpha[u, v]g(u).$$

Therefore, $\alpha[u, v]g(u) + [v, u^2] = 0$, for all $u, v \in I.$ 2.2

Replacing v by rv in equation 2.2.

$$\alpha[u, rv]g(u) + [rv, u^2] = 0.$$

$\alpha[u, rv]g(u) + [rv, u^2] = 0$, for all $u, v \quad I, r \quad R$.

$$\alpha(r)\alpha[u, v]g(u) + \alpha[u, r]\alpha(v)g(u) + r[v, u^2] + [r, u^2]v = 0. \qquad 2.3$$

Left multiplying equation 2.2 with $\alpha(r)$,

$$\alpha(r)\alpha[u, v]g(u) + \alpha(r)[v, u^2] = 0. \qquad 2.4$$

Subtracting equation 2.4 from equation 2.3,

$\alpha[u, r]\alpha(v)g(u)+r[v, u^2]v = 0$, for all $u, v \quad I, r \quad R$. 2.5

In particular if $r \in R$, say $r = w \in I$, then

$\alpha[u, w]\alpha(v)g(u) + [w, u^2]v = 0$, for all $u, v, w \quad I$. 2.6

By equation 2.2 for $u, w \in I$, we have

$$[w, u^2] = -\alpha[u, w]g(u), u, w \quad I. \qquad 2.7$$

Using equation 2.7 in 2.6,

$\alpha[u, w]\{\alpha(v)g(u) - g(u)v\} = 0$, for all $u, v, w\ I$.

Replacing v by $\alpha(v)$ in above equation, we obtain

$\alpha[u, w][\alpha(v), g(u)] = 0$, for all $u, v, w \quad I$. 2.8

Replacing w by $g(u)w$ in above equation,

$$\alpha(g(u))\alpha[u, w][\alpha(v), g(u)] + \alpha[u, g(u)]\alpha(w)[\alpha(v), g(u)] = 0.$$

Using equation 2.8 in above equation, we get

$$\alpha[u, g(u)]\alpha(w)[\alpha(v), g(u)] = 0.$$

Replacing $\alpha(g(u))$ by $g(u)$ and v by u in above equation,

$[\alpha(u), g(u)]I[\alpha(u, g(u))] = 0$, for all $u \in I$.

In view of lemma 2.1, I is a semiprime ring, so that

$[\alpha(u), g(u)] = 0$, for all $u \in$ I. or $[g(u), \alpha(u)] = 0$, for all $u \in I$.

By using similar approach we can prove the same result for $G[u, v] = -(uv + uv)$, for all $u, v \in I$.

Theorem 1.2 : R is a semiprime ring, I is a nonzero ideal of R and (G, g) is a generalized skew reverse derivation associated with an automorphism α such that $G(u \square v) = \pm (uv + vu)$, for all $u, v\ I$, then $[g(u), \alpha(u)] = 0$, for all $u \in I$.

Proof: Firstly, consider $G(u \square v) = uv + vu$, for all $u, v \quad I$, 2.9

Replacing v by uv in equation 2.9, then $G(u \square uv) = uuv + uvu$, for all $u, v \quad I$.

$$u^2v + uvu = G\{u(u \square uv)\}.$$

$$= G(u \square v)u + \alpha(u \square v)g(u).$$

$= uvu + vu^2 + \alpha(u \square v)g(u).$

Therefore $\alpha(u \square v)g(u) + [v, u^2] = 0$, for all $u, v \in I$. 2.10

Replacing v by rv in equation 2.10

$\alpha(u \square rv)g(u) + [rv, u^2] = 0$, for all $u, v \in I, r \in R$.

$\alpha\{r(u \square v) + [u, r]v\}g(u) + [rv, u^2] = 0.$

$\alpha(r)\alpha(u \square v)g(u) + \alpha[u,r]\alpha(v)g(u) + r[v, u^2] + [r, u^2]v = 0.$ 2.11

Left multiplying equation 2.10 with $\alpha(r)$.

$\alpha(r)\alpha(u \square v)g(u) + \alpha(r)[v, u^2] = 0.$ 2.12

Subtracting equation 2.12 from equation 2.11,

$\alpha[u, r]\alpha(v)g(u) + r[v, u^2] + [r, u^2]v - \alpha(r)[v, u^2] = 0.$

Replacing $\alpha(r)$ by r in above equation.

$\alpha[u, r]\alpha(v)g(u) + r[v, u^2]\, v = 0$, for all $u, v \quad I, r \quad R$. 2.13

The equation 2.13 is same as equation 2.5 in theorem 2.1. Thus, by same argument as in theorem 2.1, we conclude the result. By using similar approach we can prove the same result for $G(u \square v) = -(uv + vu)$ for all $u, v \;\; I$.

Theorem 1.3 : R is a semiprime ring. I is a nonzero ideal of R and (G, g) is a generalized skew reverse derivation associated with an automorphism α such that $G(uv) \pm (u \square v) = 0$, for all $u, v \;\; I$, then R is commutative.

Proof: Firstly, consider $G(uv) - (u \square v) = 0$, for all $u, v \quad I$, 2.14

Replacing u by wu in equation 2.14, then

$= G(uv)w + \alpha(uv)g(w) - (wuv + vwu) = 0$, for all $u, v, w \quad I$.

Adding and subtracting $(u \square v)w$ in right side of the above equation,

$G(uv)w + \alpha(uv)g(w) - (wuv + vwu) + (u \square v)w - (u \square v)w = 0.$

$\{G(uv) - (u \square v)\}w + \alpha(uv)g(w) - wuv - vwu + uvw + vuw = 0.$

Using equation 2.14 in above equation,

$\alpha(\underline{uv})g(w) + [uv, w] + v[u, w] = 0.$ 2.15

Replacing v by uv in equation 2.15, then

$\alpha(u^2v)g(w) + [u^2v, w] + uv[u, w] = 0.$ 2.16

Left multiplying equation 2.15 by $\alpha(u)$, we get

$\alpha(u^2v)g(w) + \alpha(u)[uv, w] + \alpha(u)v[u, w] = 0$, for all $u, v, w \in I$. 2.17

Subtracting equation 2.17 from equation 2.16 and replacing $\alpha(u)$ by u, we get

$0 = [u^2, v, w] - u[uv, w].$

$= u^2vw - wu^2v - u^2vw + uwuv.$

$= [u, w]uv.$ 2.18

Replacing w by wz, $z \in I$, in equation 2.18, then we get $[u, wz]uv = 0$.

$w[u, z]uv + [u, w]zuv = 0.$ 2.19

Using equation 2.18 by replacing w by zw,

$[u, w]zuv = 0.$ 2.20

Again replacing z by zw in equation 2.20

$[u, w]zwuv = 0$, for all $u, v, w, z \in I$. 2.21

Replacing v by wv in equation 2.20,

$[u, w]zuwv = 0$, for all $u, v, w, z \in I$. 2.22

Ubtracting equation 2.21 from equation 2.22,

$[u, w]z[u,w]v = 0$, for all $u, v, w, z \in I$.

Replacing v by z in above eqution, we have $\{[u, w]z\}^2 = 0$.

By semiprimeness of R, we conclude that $[u, w] = 0$, for all u, w, z I. Thus R is commutative. By similar approach, we can prove the same conclusion for $G(uv) + (u \square v) = 0$, for all $u, v \in I$.

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