A comparative analysis of the transportation problem using NWCM, VAM, RAVAL'S APPROXIMATION and NEWA

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Abstract:
One of the main topics of operations research is transportation problems. The transportation model is a special class of the linear programming problem. It deals with the situation in which commodity is shipped from sources to destinations. The objective is to minimize the total shipping cost while satisfying both the supply limit and the demand requirements. Transportation problem has basic feasible solution (BFS) and then from it we obtain the optimal solution by using MODI method or stepping stone method. In this study, some methods for computing the initial feasible solution, such as North West Corner Method (NWCM), VAM (Vogel's Approximation Method), Raval's Approximation and New Approach (NEWA) have been compared.

Keywords:
North West Corner Method (NWCM), VAM (Vogel's Approximation Method), Raval's Approximation, and New Approach (NEWA)

1. Introduction

In operations, research Linear programming is one of the models of mathematical programming, which is very broad and vast. Hitchcock first proposed the transport problem (TP), which Koopmans and Kantorovich addressed separately. Monge defined and unraveled it using mathematical methods. Hitchcock developed a major transportation problem. The transportation model deals with a special class of linear programming problem in which the objective is to transport a homogeneous commodity from various origins or factories to different destinations or markets at a total minimum cost. Transportation issues are critical to logistics and supply chain management in terms of cost reduction and service improvement. Each source can supply fixed value units of goods, generally known as availability, and every destination has a fixed demand, which is called requirement. The availability of transportation problem occurs when each source can supply a constant number of output units while each destination has a fixed demand. The fundamental transportation problem in operational research involves determining the lowest cost of transporting a single article or commodity from a certain number of sources to a given number of destinations. The procedure for solving transportation problems consists of formulating a mathematical model of transportation problems, finding an initial basic feasible then optimizing the initial basic feasible solution obtained by using the Modified Distribution (MODI) or Stepping Stone methods.

2. General Mathematical Model of Transportation Problem:

Let there be *m* sources of supply, $S_1, S_2, ..., S_m$ having $a_i\ (i = 1,2,...,m)$ units of supply (or capacity), respectively to be transported to n destinations, $D_1, D_2, ..., D_n$ with $b_j\ (j = 1,2,...,n)$ units of demand (or requirement), respectively. Let c_{ij} be the cost of shipping one unit of the commodity from source *i* to destination *j*. If x_{ij} represents number of units shipped from source *i* to destination *j*, the problem is to determine the transportation schedule so as to minimize the total transportation cost while satisfying the supply and demand conditions. Mathematically, the transportation problem, in general, may be stated as follows:

minimize (total cost) $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ -------(1)

subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,...,m$ (supply constraints) -------(2)

$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,...,n$ (demand constraints) -------(3)

and $x_{ij} \geq 0$ for all *i* and *j*.

From \ To	D_1	D_2	…	D_n	Supply (a_i)
S_1	c_{11} x_{11}	c_{12} x_{12}	…	c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}	…	c_{2n} x_{2n}	a_2
⋮	⋮	⋮		⋮	⋮
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}	…	c_{mn} x_{mn}	a_m
Demand (b_j)	b_1	b_2	…	b_n	

There are two types of transportation problem namely Balanced Transportation Problem (BTP) and Unbalanced Transportation Problem(UTP). The transportation problem is said to be balanced transportation problem if total number of supply is same as total number of demand *i.e.* $\sum_{i-1}^{m} a_i = \sum_{j-1}^{n} b_j$ and it is said to be unbalanced transportation problem if total number of supply is not same as total number of demand *i.e.* $\sum_{i-1}^{m} a_i \neq \sum_{j-1}^{n} b_j$.

3. Methods of Finding Initial Solution

3.1 North West Corner Rule

Start from the left hand side top corner or cell and make allocations depending on the availability and requirement constraint. If the availability constraint is less than the requirement constraint, then for that cell make allocation in units which is equal to the availability constraint. In general, verify which is the smallest among the availability and requirement and allocate the smallest one to the cell under question. Then proceed allocating either sidewise or downward to satisfy the rim requirement. Continue this until all the allocations are over. Once all the allocations are over, i.e., both rim requirement (column and row i.e., availability and requirement constraints) are satisfied, write allocations.

3.2 Vogel's Approximation Method (VAM)

In this method, an allocation is made on the basis of the opportunity (or penalty or extra) cost that would have been incurred if the allocation in certain cells with minimum unit transportation cost were missed. Hence, allocations are made in such a way that the penalty cost is minimized. The steps of VAM are as follows:

Step 1: Calculate the penalties for each row (column) by taking the difference between the smallest and next smallest unit transportation cost in the same row (column). This difference indicates the penalty or extra cost that has to be paid if decision-maker fails to allocate to the cell with the minimum unit transportation cost.

Step 2: Select the row or column with the largest penalty and allocate as much as possible in the cell that has the least cost in the selected row or column and satisfies the rim conditions. If there is a tie in the values of penalties, it can be broken by selecting the cell where the maximum allocation can be made.

Step 3: Adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of them is crossed out and the remaining row (column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step 4: Repeat Steps 1 to 3 until the available supply at various sources and demand at various destinations is satisfied.

3.3 Raval's Approximation Method

The steps of Raval's Approximation method are as follows:

Step-1: The number of supply and demand should be equal, and metric should have same number of row and column, if it is not then adding dummy column or row in it.

Step-2: Find the number of basic variables by (m + n −1) & select lowest number in the cell accordingly

Step-3: After selecting a number of basic variables (m + n − 1) and multiply the lowest element from the demand or supply to the corresponding cell.
Step-4: For balancing the demand and supply, whatever we multiply with the cells that should be substitute with corresponding demand or supply. Further, select the second lowest element from the demand or supply and the process continues further.
Step-5: The operation continues until we do not reach the number of basic variables and utilization of all demand and supply.
Step-6: There should not be any negative elements in cells, demand, and supply, or it violates the basic rule of transportation model.
Step-7: Multiplication and submission of cells with corresponding demands and supply forgetting initial basic feasible solution

3.4 New Approach (NEWA)Method

The steps of New Approach (NEWA) method are as follows:
Step 1: Formulate the Transportation Cost Matrix. If the problem is unbalanced, make it a balanced problem by introducing a dummy source or a dummy destination accordingly.
Step 2: Identify the cell for allocation which has the least unit transportation cost (c_{ij}) in each row and columns.
Step 3: If the least cost of any row is the least cost of any column, then select those least costs.
Step 4: Crossed off the rows and columns of the least costs obtained in Step 3.
Step 5: Repeat steps 2 to 4 for uncrossed rows and columns until at least one cell is marked in each row and each column
Step 6: First allocate min (ai, bj) amount of units without violating the demand and supply to the cell of the least cost in the priority with the above step and update the supply and demand
Step 7: Repeat Step 6 for the next least cost and continue until all the selected cells run out.
Step 8: Extract the initial feasible solution

4. Numerical Examples

Example 4.1

	D_1	D_2	D_3	D_4	Supply
P_1	10	16	13	22	16
P_2	12	13	11	15	14
P_3	15	18	14	10	18
P_4	17	21	15	19	9
Demand	12	13	15	17	

Solution: Sum of Supply=Sum of Demand =57, Balanced Transportation Problem
(i) Initial feasible solution by North West Corner Method

	D_1	D_2	D_3	D_4	Supply
P_1	10(12)	16(4)	13	22	16
P_2	12	13(9)	11(5)	15	14
P_3	15	18	14(10)	10(8)	18
P_4	17	21	15	19(9)	9
Demand	12	13	15	17	

Transportation Cost $=10\times12+16\times4+13\times9+11\times5+14\times10+10\times8+19\times9=$ Rs.747

(ii) Initial feasible solution by VAM

	D_1	D_2	D_3	D_4	Supply
P_1	10(12)	16	13(4)	22	16
P_2	12	13(13)	11(1)	15	14
P_3	15	18	14(1)	10(17)	18
P_4	17	21	15(9)	19	9
Demand	12	13	15	17	

Transportation Cost
$=10\times12+13\times4+13\times13+11\times1+14\times1+10\times17+15\times9=671$

(iii) Initial feasib**le** solution by Raval's Approximation Method

	D_1	D_2	D_3	D_4	Supply
P_1	10(4)	16(12)	13	22	16
P_2	12(8)	13	11(6)	15	14
P_3	15	18(1)	14	10(17)	18
P_4	17	21	15(9)	19	9
Demand	12	13	15	17	

Transportation Cost $=10\times4+16\times12+12\times8+11\times6+18\times1+10\times17+15\times9=$ Rs.717

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	Supply
P_1	10(12)	16(3)	13(1)	22	16
P_2	12	13	11(14)	15	14
P_3	15	18(1)	14	10(17)	18
P_4	17	21(9)	15	19	9
Demand	12	13	15	17	

Transportation Cost $=10\times12+16\times3+13\times1+11\times14+18\times1+10\times17+21\times9=$ Rs.712
(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	Supply	
P_1	10(12)	16 $\boxed{1}$	13(4)	22 $\boxed{13}$	16	$u_1=13$
P_2	12 $\boxed{4}$	13(13)	11(1)	15 $\boxed{8}$	14	$u_2=11$
P_3	15 $\boxed{4}$	18 $\boxed{2}$	14(1)	10(17)	18	$u_3=14$
P_4	17 $\boxed{5}$	21 $\boxed{4}$	15(9)	19 $\boxed{8}$	9	$u_4=15$
Demand	12	13	15	17		
	$v_1=3$	$v_2=2$	$v_3=0$	$v_4=-4$		

Optimal Transportation Cost$=10\times12+13\times13+11\times1+14\times1+10\times17+15\times9=$Rs.671

Example 4.2

	D_1	D_2	D_3	D_4	Supply
P_1	4	6	9	5	16
P_2	2	6	4	1	12
P_3	5	7	2	9	15
Demand	12	14	9	8	

Solution: Sum of Supply=Sum of Demand =43, Balanced Transportation Problem
(i) Initial feasible solution by North West Corner Method (NWCM)

	D_1	D_2	D_3	D_4	Supply
P_1	4(12)	6(4)	9	5	16
P_2	2	6(10)	4(2)	1	12
P_3	5	7	2(7)	9(8)	15
Demand	12	14	9	8	

Transportation Cost $=4\times12+6\times4+6\times10+4\times2+2\times7+9\times8$ = Rs.202
(ii) Initial feasible solution by VAM

	D_1	D_2	D_3	D_4	Supply
P_1	4(8)	6(8)	9	5	16
P_2	2(4)	6	4	1(8)	12
P_3	5	7(6)	2(9)	9	15

Demand	12	14	9	8

Transportation Cost $= 4\times 8 + 6\times 8 + 2\times 4 + 1\times 8 + 7\times 6 + 2\times 9 =$ Rs.156

(iii) Initial feasible solution by Raval's Approximation

	D_1	D_2	D_3	D_4	Supply
P_1	4(8)	6(8)	9	5	16
P_2	2(4)	6	4	1(8)	12
P_3	5	7(6)	2(9)	9	15
Demand	12	14	9	8	

Transportation Cost $= 4\times 8 + 6\times 8 + 2\times 4 + 1\times 8 + 7\times 6 + 2\times 9 =$ Rs.156

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	Supply
P_1	4(8)	6(8)	9	5	16
P_2	2(4)	6	4	1(8)	12
P_3	5	7(6)	2(9)	9	15
Demand	12	14	9	8	

Transportation Cost $= 4\times 8 + 6\times 8 + 2\times 4 + 1\times 8 + 7\times 6 + 2\times 9 =$ Rs.156

(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	Supply	
P_1	4(8)	6(8)	9 $\boxed{8}$	5 $\boxed{2}$	16	$u_1=0$
P_2	2(4)	6 $\boxed{2}$	4 $\boxed{5}$	1(8)	12	$u_2=-2$
P_3	5 $\boxed{0}$	7(6)	2(9)	9 $\boxed{5}$	15	$u_3=1$
Demand	12	14	9	8		
	$v_1=4$	$v_2=6$	$v_3=1$	$v_4=3$		

Optimal Transportation Cost $= 4\times 8 + 6\times 8 + 2\times 4 + 1\times 8 + 7\times 6 + 2\times 9 =$ Rs.156

Example 4.3

	D_1	D_2	D_3	D_4	Supply
P_1	7	5	14	16	12
P_2	5	10	17	19	8
P_3	8	6	12	15	15
P_4	6	12	10	11	14
P_5	10	15	8	14	11
Demand	12	13	14	15	

Solution: Sum of Supply=60, Sum of Demand =54,

Unbalanced Transportation Problem. Introduce Dummy Column D_m

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	7	11	14	16	0	12
P_2	5	10	17	19	0	8
P_3	8	6	12	15	0	15
P_4	6	12	10	11	0	14
P_5	10	15	8	14	0	11
Demand	12	13	14	15	6	

(i) Initial feasible solution by North West Corner Method

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	7(12)	11	14	16	0	12
P_2	5	10(8)	17	19	0	8
P_3	8	6(5)	12(10)	15	0	15
P_4	6	12	10(4)	11(10)	0	14
P_5	10	15	8	14(5)	0(6)	11

Demand	12	13	14	15	6

Transportation Cost $= 7\times12+10\times8+6\times5+12\times10+10\times4+11\times10+14\times5+0\times6$
$= \text{Rs.}534$

(ii) Initial feasible solution by VAM

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	7(4)	11	14(7)	16(1)	0	12
P_2	5(8)	10	17	19	0	8
P_3	8	6(13)	12(2)	15	0	15
P_4	6	12	10	11(14)	0	14
P_5	10	15	8(5)	14	0(6)	11
Demand	12	13	14	15	6	

Transportation Cost $= 7\times4+14\times7+16\times1+5\times8+6\times13+12\times2+11\times14+8\times5+0\times6$
$= \text{Rs.}368$

(iii) Initial feasible solution by Raval's Approximation

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	7	11(6)	14	16	0(6)	12
P_2	5(8)	10	17	19	0	8
P_3	8	6(7)	12	15(8)	0	15
P_4	6(4)	12	10(10)	11	0	14
P_5	10	15	8(4)	14(7)	0	11
Demand	12	13	14	15	6	

Transportation Cost $=11\times6+0\times6+5\times8+6\times7+15\times8+6\times4+10\times10+8\times4+14\times7$
$= \text{Rs.}522$

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	7(4)	11	14	16(4)	0(6)	12
P_2	5(8)	10	17	19	0	8
P_3	8	6(13)	12	15(2)	0	15
P_4	6	12	10(3)	11(11)	0	14
P_5	10	15	8(11)	14	0	11
Demand	12	13	14	15	6	

Transportation Cost $= 7\times4+5\times8+6\times13+10\times3+8\times11+16\times4+15\times2+11\times11+0\times6$
$= \text{Rs.}449$

(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	D_m	Supply	
P_1	7(4)	11 $\boxed{3}$	14(1)	16(1)	0(6)	12	u_1=0
P_2	5(8)	10 $\boxed{4}$	17 $\boxed{5}$	19 $\boxed{6}$	0 $\boxed{2}$	8	u_2=-2
P_3	8 $\boxed{3}$	6(13)	12(2)	15 $\boxed{2}$	0 $\boxed{2}$	15	u_3=-2
P_4	6 $\boxed{4}$	12 $\boxed{4}$	10 $\boxed{1}$	11(14)	0 $\boxed{5}$	14	u_4=-5
P_5	10 $\boxed{9}$	15 $\boxed{3}$	8(11)	14 $\boxed{5}$	0 $\boxed{6}$	11	u_5=-6
Demand	12	13	14	15	6		
	v_1=7	v_2=8	v_3=14	v_4=16	v_5=0		

Optimal Transportation Cost $= 7\times4+14\times1+16\times1+0\times6+5\times8+6\times13+12\times2+11\times14+8\times11$
$= \text{Rs.}354$

Example 4.4

	D_1	D_2	D_3	D_4	Supply
P_1	4	7	11	15	4
P_2	6	5	6	12	6
P_3	8	12	4	7	5
P_4	5	8	10	9	9
P_5	10	6	13	4	7
Demand	7	8	6	10	

Solution: Sum of Supply=Sum of Demand =31,
Balanced Transportation Problem

(i) Initial basic solution by North west Corner Method

	D_1	D_2	D_3	D_4	Supply
P_1	4(4)	7	11	15	4
P_2	6(3)	5(3)	6	12	6
P_3	8	12(5)	4	7	5
P_4	5	8	10(6)	9(3)	9
P_5	10	6	13	4(7)	7
Demand	7	8	6	10	

Transportation Cost $= 4\times4+6\times3+5\times3+12\times5+10\times6+9\times3+4\times7$
=Rs.224

(ii) Initial feasible solution by VAM

	D_1	D_2	D_3	D_4	Supply
P_1	4(4)	7	11	15	4
P_2	6	5(5)	6(1)	12	6
P_3	8	12	4(5)	7	5
P_4	5(3)	8(3)	10	9(3)	9
P_5	10	6	13	4(7)	7
Demand	7	8	6	10	

Transportation Cost $= 4\times4+5\times5+6\times1+4\times5+5\times3+8\times3+9\times3+4\times7$
=Rs.161

(iii) Initial feasible solution by Raval's method

	D_1	D_2	D_3	D_4	Supply
P_1	4(4)	7	11	15	4
P_2	6	5(5)	6(1)	12	6
P_3	8	12	4(5)	7	5
P_4	5(3)	8	10	9(6)	9
P_5	10	6(3)	13	4(4)	7
Demand	7	8	6	10	

Transportation Cost $= 4\times4+5\times5+6\times1+4\times5+5\times3+9\times6+6\times3+4\times4$
=Rs.170

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	Supply
P_1	4(4)	7	11	15	4
P_2	6	5(6)	6	12	6
P_3	8	12	4(5)	7	5
P_4	5(3)	8	10(1)	9(5)	9
P_5	10	6(2)	13	4(5)	7
Demand	7	8	6	10	

Transportation Cost $= 4\times4+5\times6+4\times5+5\times3+10\times1+9\times5+6\times2+4\times5$
=Rs.168

(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	Supply	
P_1	4(4)	7 $\boxed{0}$	11 $\boxed{3}$	15 $\boxed{7}$	4	u_1=-1
P_2	6 $\boxed{4}$	5(5)	6(1)	12 $\boxed{6}$	6	u_2=-3
P_3	8 $\boxed{8}$	12 $\boxed{9}$	4(5)	7 $\boxed{3}$	5	u_3=-5
P_4	5(3)	8(3)	10 $\boxed{1}$	9(3)	9	u_4=0
P_5	10 $\boxed{2}$	6 $\boxed{3}$	13 $\boxed{9}$	4(7)	7	u_5=-5
Demand	7	8	6	10		
	v_1=5	v_2=8	v_3=9	v_4=9		

Optimal Transportation Cost $= 4\times4+5\times5+6\times1+4\times5+5\times3+8\times3+9\times3+4\times7$
=Rs.161

Example 4.5

	D_1	D_2	D_3	D_4	Supply
P_1	10	18	20	19	23
P_2	15	20	11	16	14
P_3	13	17	18	15	19
P_4	14	12	22	21	24
Demand	17	20	22	21	

Solution: Sum of Supply=Sum of Demand =80, Balanced Transportation Problem

(i) Initial feasible solution by North West Corner Rule

	D_1	D_2	D_3	D_4	Supply
P_1	10(17)	18(6)	20	19	23
P_2	15	20(14)	11	16	14
P_3	13	17	18(19)	15	19
P_4	14	12	22(3)	21(21)	24
Demand	17	20	22	21	

Transportation Cost
$=10\times17+18\times6+20\times14+18\times19+22\times3+21\times21$=Rs.1407

(ii) Initial feasible solution by VAM

	D_1	D_2	D_3	D_4	Supply
P_1	10(17)	18	20(4)	19(2)	23
P_2	15	20	11(14)	16	14
P_3	13	17	18	15(19)	19
P_4	14	12(20)	22(4)	21	24
Demand	17	20	22	21	

Transportation Cost
$=10\times17+20\times4+19\times2+11\times14+15\times19+12\times20+22\times4$ =Rs.1055

(iii) Initial feasible solution by Raval's Approximation Method

	D_1	D_2	D_3	D_4	Supply
P_1	10(6)	18(17)	20	19	23
P_2	15	20	11(14)	16	14
P_3	13(11)	17	18(8)	15	19
P_4	14	12(3)	22	21(21)	24

Demand	17	20	22	21

Transportation Cost
$=10\times6+18\times17+11\times14+13\times11+18\times8+12\times3+21\times21$=Rs.1284

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	Supply
P_1	10(17)	18	20(4)	19(2)	23
P_2	15	20	11(14)	16	14
P_3	13	17	18	15(19)	19
P_4	14	12(20)	22(4)	21	24
Demand	17	20	22	21	

Transportation Cost
$=10\times17+20\times4+19\times2+11\times14+15\times19+12\times20+22\times4$=Rs.1055

(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	Supply	
P_1	10(17)	18 $\boxed{8}$	20(4)	19(2)	23	u_1=0
P_2	15 $\boxed{14}$	20 $\boxed{19}$	11(14)	16 $\boxed{6}$	14	u_2=-9
P_3	13 $\boxed{7}$	17 $\boxed{11}$	18 $\boxed{2}$	15(19)	19	u_3=-4
P_4	14 $\boxed{2}$	12(20)	22(4)	21 $\boxed{4}$	24	u_4=2
Demand	17	20	22	21		
	v_1=10	v_2=10	v_3=20	v_4=19		

Optimal Transportation Cost$=10\times17+20\times4+19\times2+11\times14+15\times19+12\times20+22\times4$
=Rs.1055

<u>Example 4.6</u>

	D_1	D_2	D_3	D_4	Supply
P_1	4	6	8	13	20
P_2	10	5	5	11	30
P_3	12	16	15	10	15
P_4	3	9	14	14	13
Demand	6	8	18	6	

Solution: Sum of Supply=78, Sum of Demand =38, Unbalanced Transportation Problem
Introduce Dummy Column

(i) Initial basic feasible solution by North West Corner Method

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	4(6)	6(8)	8(6)	13	0	20
P_2	10	5	5(12)	11(6)	0(12)	30
P_3	12	16	15	10	0(15)	15
P_4	3	9	14	14	0(13)	13
Demand	6	8	18	6	40	

Transportation Cost $=4\times6+6\times8+8\times6+5\times12+11\times6+0\times12+0\times15+0\times13$=Rs.246

(ii) Initial basic feasible solution by VAM

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	4	6(1)	8(13)	13(6)	0	20
P_2	10	5	5(5)	11	0(25)	30
P_3	12	16	15	10	0(15)	15
P_4	3(6)	9(7)	14	14	0	13
Demand	6	8	18	6	40	

Transportation Cost $=6\times1+8\times13+13\times6+5\times5+0\times25+0\times15+3\times6+9\times7$ =Rs.294

(iii) Initial feasible solution by Raval's Approximation

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	4	6	8	13	0(20)	20
P_2	10	5(1)	5(9)	11	0(20)	30
P_3	12	16	15(9)	10(6)	0	15
P_4	3(6)	9(7)	14	14	0	13
Demand	6	8	18	6	40	

Transportation Cost $= 0\times 20 + 5\times 1 + 5\times 9 + 0\times 20 + 15\times 9 + 10\times 6 + 3\times 6 + 9\times 7$
=Rs.326

(iv) Initial feasible solution by New Approach (NEWA)Method

	D_1	D_2	D_3	D_4	D_m	Supply
P_1	4(6)	6	8	13	0(14)	20
P_2	10	5(8)	5(18)	11	0(4)	30
P_3	12	16	15	10(6)	0(9)	15
P_4	3	9	14	14	0(13)	13
Demand	6	8	18	6	40	

Transportation Cost $= 4\times 6 + 0\times 14 + 5\times 8 + 5\times 18 + 0\times 4 + 10\times 6 + 0\times 9 + 0\times 13$
=Rs.214

(v) Optimal Solution by MODI

	D_1	D_2	D_3	D_4	D_m	Supply	
P_1	4 $\boxed{1}$	6 $\boxed{1}$	8 $\boxed{3}$	13 $\boxed{5}$	0(20)	20	$u_1=0$
P_2	10 $\boxed{7}$	5(8)	5(18)	11 $\boxed{1}$	0(4)	30	$u_2=0$
P_3	12 $\boxed{9}$	16 $\boxed{11}$	15 $\boxed{10}$	10(6)	0(9)	15	$u_3=0$
P_4	3(6)	9 $\boxed{4}$	14 $\boxed{19}$	14 $\boxed{4}$	0(7)	13	$u_4=0$
Demand	6	8	18	6	40		
	$v_1=3$	$v_2=5$	$v_3=10$	$v_4=19$	$v_5=0$		

Optimal Transportation Cost $= 0\times 20 + 5\times 8 + 5\times 18 + 0\times 4 + 10\times 6 + 0\times 9 + 3\times 6 + 0\times 7$ =Rs.208

5. **Comparison:** The comparisons of the results are studied in this research to measure the effectiveness of the methods. The summary of the results of the numerical example is given in Table 1.

Table 1. Comparative results of VAM, ASM. Modified-ASM, AMSTP and optimal Cost

S. N.	**Example No.**	**Method**					**Percentage of increase from the optimal cost**			
		NWCM	**VAM**	**Ravals**	**NEWA**	**Optimal Cost (MODI)**	**NWCM**	**VAM**	**Ravals**	**NEWA**
1	4.1	747	671	717	712	671	11.33	0.00	6.86	6.11
2	4.2	202	156	156	156	156	29.49	0.00	0.00	0.00
3	4.3	534	368	522	449	354	50.85	3.95	47.46	26.84
4	4.4	224	161	170	168	161	39.13	0.00	5.59	4.35
5	4.5	1407	1055	1284	1055	1055	33.36	0.00	21.71	0.00

6	4.6	246	294	326	214	208	18.27	41.35	56.73	2.88

The comparative results obtained in Table 1 are also depicted using bar graphs and the results are given in Figure 1.

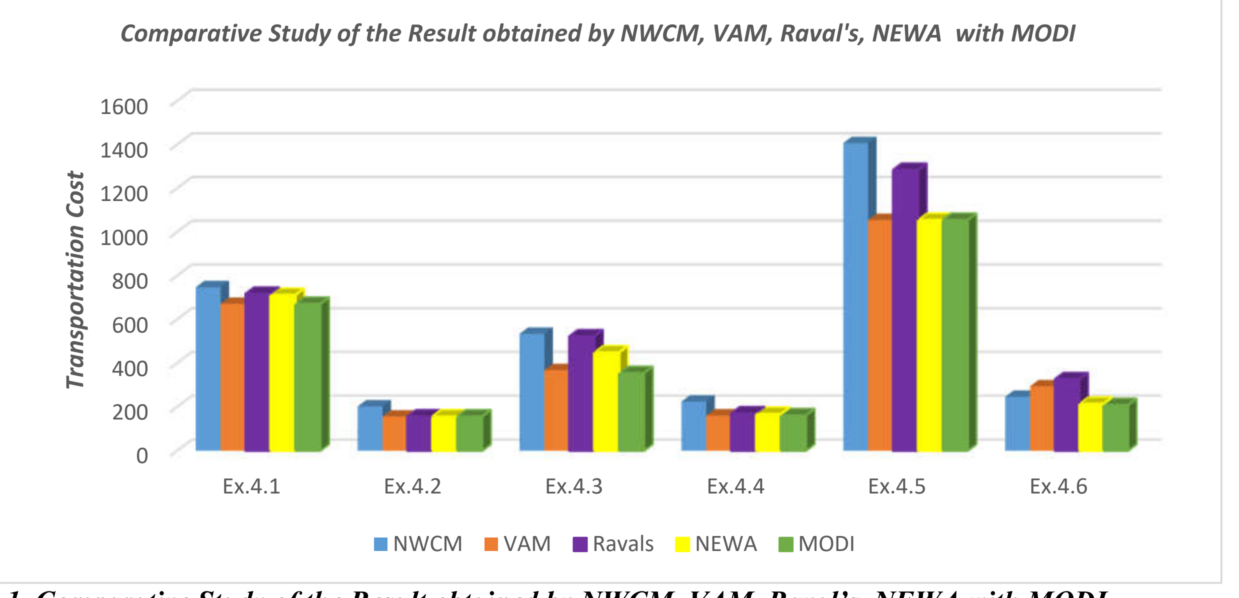

Figure 1. Comparative Study of the Result obtained by NWCM, VAM, Raval's, NEWA with MODI

Line graphs for the percentage deviation (of the NWCM, VAM, Raval's and NEWA) with optimal Cost is obtained in Table 1 are presented in Figure 2.

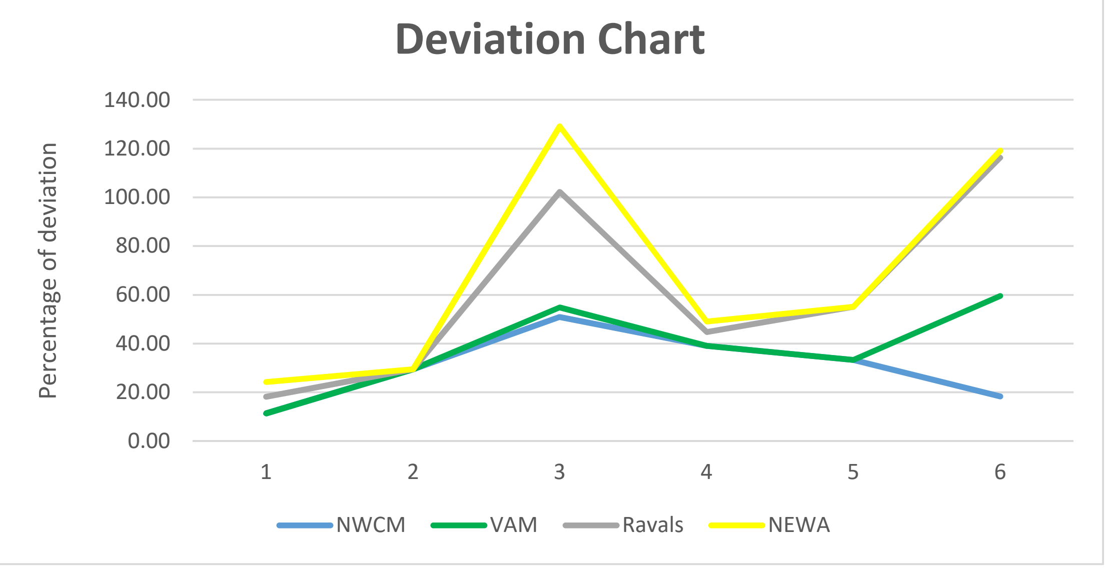

Figure 2 Percentage of Deviation of the Results obtained by the Result obtained by NWCM, VAM, Raval's, and NEWA with optimal cost

6. <u>Conclusion</u>

As seen from the Table -1, Figure-1 and Figure-2, VAM method is more effective than NWCM, Raval's and NEWA. Optimal cost of transportation problem comes quickly from VAM. NEWA method is more efficient then NWCR and Raval's approximation method

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